Sliding Mode Control

Part 4: Noncontinuous Control Theory

https://xsro.github.io/print/Control-for-Integrator-Systems-4.pdf

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Contents

Bibliography

CORTES J, 2008. Discontinuous Dynamical Systems[J/OL]. IEEE Control Systems Magazine, 28(3): 36-73. DOI:10.1109/MCS.2008.919306.

1 Discontinuous System Theory

see (Cortes, 2008)

1.1 Ternary Differential Equations' Solutions

Table 1: solutions to ternary differential equations

differetial equation	**differetial inclusion**	**classical solution**	**caratheodory solution**	**Filippov solution**
$\dot{x} = \begin{cases} 1 & \text{if } x<0 \\ a & \text{if } x=0 \\ -1 & \text{if } x>0 \end{cases}$	$\dot{x} \in \mathcal{F}(x) = \begin{cases} 1 & \text{if } x<0 \\ [-1,1] & \text{if } x=0 \\ -1 & \text{if } x>0 \end{cases}$	Only when $a = 0$, classical solution exists. The maximal classical solution is 1. if $x(0) > 0$, $x_1(t) = x(0) - t, t < x(0)$ 2. if $x(0) < 0$, $x_2(t) = x(0) + t, t < -x(0)$ 3. if $x(0) = 0$, $x_3(t) = 0, t \in [0, \infty)$	Only when $a = 0$, caratheodory solution exists. The maximal classical solution is 1. if $x(0) > 0$, $x_1(t) = \max(x(0) - t, 0), t \in [0, \infty)$ 2. if $x(0) < 0$, $x_2(t) = \min(x(0) + t, 0), t \in [0, \infty)$ 3. if $x(0) = 0$, $x_3(t) = 0, t \in [0, \infty)$ **Note**:These only absolutely continuous (not continuously differentiable)	Whatever the value of a is, the Filippov solution is 1. if $x(0) > 0$, $x_1(t) = \max(x(0) - t, 0), t \in [0, \infty)$ 2. if $x(0) < 0$, $x_2(t) = \min(x(0) + t, 0), t \in [0, \infty)$ 3. if $x(0) = 0$, $x_3(t) = 0, t \in [0, \infty)$
$\dot{x} = \begin{cases} -1 & \text{if } x<0 \\ a & \text{if } x=0 \\ 1 & \text{if } x>0 \end{cases}$	$\dot{x} \in \mathcal{F}(x) = \begin{cases} -1 & \text{if } x<0 \\ [-1,1] & \text{if } x=0 \\ 1 & \text{if } x>0 \end{cases}$	From $x = x(0) \neq 0$,classical solution exists as 1. $x_1(t) = x(0) + t$ if $x(0) > 0$ 2. $x_2(t) = x(0) - t$ if $x(0) < 0$ From $x = x(0) = 0$, classical solution exists when $a = 1$ or $a = -1$ 1. when $a = 1$, $x_1(t) = t, t \in [0, \infty)$ 2. when $a = -1$,$x_2(t) = -t, t \in [0, \infty)$	From $x = x(0) \neq 0$,classical solution exists as 1. $x_1(t) = x(0) + t$ if $x(0) > 0$ 2. $x_2(t) = x(0) - t$ if $x(0) < 0$. From $x = x(0) = 0$, two caratheodory solutions exist for **all** $a \in \mathbb{R}$ 1. $x_1(t) = t, t \in [0, \infty)$ 2. $x_2(t) = -t, t \in [0, \infty)$ These two solutions only violate the vector field in $t = 0$	Filippov solution exists for all $a \in \mathbb{R}$ and $x(0) \in \mathbb{R}$. 1. if $x(0) \geq 0$, $x_1(t) = x(0) + t, t \in [0, \infty)$ 2. if $x(0) \leq 0$,, $x_2(t) = x(0) - t, t \in [0, \infty)$ **Note**: When $x(0) = 0$, exists two Filippov solutions.
$\dot{x} = \begin{cases} 1 & \text{if } x\neq0 \\ 0 & \text{if } x=0 \end{cases}$	$\dot{x} \in \{1\}$	$x = 0, t \in [0, \infty)$	two caratheodory solutions: 1. $x(t) = 0, t \in [0, \infty)$ 2. $x(t) = t, t \in [0, \infty)$	one unique solution: 1. $x(t) = t, t \in [0, \infty)$

1.2 Conditions for Existence and Uniqueness of Classical, Caratheodory, Filippov Solutions

Table 2: conditions of solutions to $\dot{x} = X(x(t))$

	solution	existence	uniqueness
classical	continuously differentiable	$X : \mathbb{R}^d \to \mathbb{R}^d$ is continuous	essentially one-sided Lipschitz on $B(x, \varepsilon)$,[1]
Filippov	absolutely continuous	$X : \mathbb{R}^d \to \mathbb{R}^d$ is measurable and locally essentially bounded	Prosition 4&5

Proposition 4

Let $X : \mathbb{R}^d \to \mathbb{R}^d$ be measurable and locally essentially bounded. Assume that, for all $x in \mathbb{R}^d$, there exists $\epsilon > 0$ such that X is essentially one-sided Lipschitz on $B(x, \epsilon)$. Then, for all $x_0 \in \mathbb{R}^d$, there exists a unique Filippov solution of (10) with initial condition $x(0) = x_0$.

Proposition 5

Let $X : \mathbb{R}^d \to \mathbb{R}^d$ be a piecewise continuous vector field, with $\mathbb{R}^d = D_1 \cup D_2$. Let $S_X = \mathrm{bdry}(D_1) = \mathrm{bdry}(D_2)$ be the set of points at which X is discontinuous, and assume that S_X is a C^2-manifold. Furthermore, assume that, for $i \in \{1, 2\}$, $X|_{\overline{D_i}}$ is continuously differentiable on D_i and $X|_{\overline{D_1}} - X|_{\overline{D_2}}$ is continuously differentiable on S_X. If, for each $x \in S_X$, either $X|_{\overline{D_1}}(x)$ points into D_2 or $X|_{\overline{D_2}}(x)$ points into D_1, then there exists a unique Filippov solution of (10) starting from each initial condition.

[1] Every vector field that is locally Lipschitz at x satisfies the one-sided Lipschitz condition on a neighborhood of x, but the converse is not true.

THANKS